NUMERICAL PREDICTION OF FROST THICKNESS GROWTH OVER A COLD CYLINDER

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Abstract. This paper evaluates numerically some of the parameters involved in modeling of the frost process formation over a cold cylinder surface subject to humid air flow. A numerical modeling to predict frosting grow process was utilized. The model employs one-dimensional transient formulation based upon the local volume averaging technique. Empirical Nusselt correlations from the literature, obtained in experimental studies on frost formation phenomena of frost grow over a cold cylinder surface were utilized. A new correlation for Nusselt number based on the experimental correlation of Kim was estimated. For the new Nusselt correlation an optimization method that adjusts the numerical solution for frost grows prediction with the experimental results of the frost layer thickness was used. The calculation procedure allows the estimation of an adjusted parameter $K$ in the empirical Nusselt correlation. The modeling process was validated by comparison with available experimental data.

Keywords: Frost, Cylinder, Numerical simulation, Nusselt correlations.
1 INTRODUCTION

When humid air comes into contact with a cold surface whose temperature is below the dew point temperature of water vapor in air condensation will occur, if the surface temperature is also below freezing frost will form. The deposition of frost is normally accompanied by reduction in the heat exchanger efficiency because of an associated reduction in the heat transfer rate as well as an eventual blockage of the air passages. At the initial stages of the deposition process, a temporary increase in the heat transfer rate is surface acts as finned surfaces, thus enhancing heat transfer.

Frost formation over cold surfaces may lead to serious operational problems in, e.g., compressor blades and wings or may impair the thermal performance as in the case of refrigeration equipment. As the frost layer grows thicker, the heat transfer is affected in part because of the insulating effect of the frost layer. This can adversely affect the performance of the cooling coils of commercial and industrial equipment. To facilitate the design tasks and the selection of the defrost cycles of such cooling equipment, it is necessary to have deep understanding of the frost formation process both theatrically and experimentally as well as in real operation conditions Yang and Lee (2004).

The frost formation process is complex due to the fact that the frost properties vary continuously during the development of the frost layer and also because of the continuous changes of the temporal and spatial air-frost interface temperature. As the interface temperature change, the partial pressure of the vapor at the surface also change, leading to a change in the thermal and diffusion boundary layers and the frost growth rates.

Based on experimental observations, frost formation periods have been described by Hayashi et.al (1977) in forced convection condition. They observed that frost formation consist of two periods. During the first period frost can be simulated as a forest of ice column until a transition time, after which the frost may be considered as a homogenous porous medium. The characterization of early-stage of frost growth and estimation of transition time are essential for any numerical modeling. Tao et al. (1992) represented a mathematical model subdivided the process of frost formation into two basic stages, the first is the one-dimensional growth of ice columns and the second is the multidimensional growth of porous structure. Ismail et al. (1998) described a two-dimensional model to study frost growth around a cylinder in a wet stream and evaluated local properties during the frost formation stages. Tahavvor & Yaghoubi (1977) investigated the mathematical model to predict the transition time and frost properties in natural convection of frost formation.

The effect of frost formation on a cold surface has been studied for many years, and many publications have concentrated on the experimental and numerical investigation of frost growing under free and forced convection. Previous researches have shown that the properties of the frost are mostly influenced by environmental parameters such as the air temperature, humidity, velocity, geometric and cold surface temperature.

Frost studies for forced convection are extensive Raju and Sherif (1993), Ismail et al. (1998), Lee and Ro (2002) and Sengupta et al. (1998) presented numerical and experimental models to predict frosting process over a circular cylinder subjected to a humid air cross flow. The temperature distribution, the thickness of the frost layer, and the temperature distribution in the cylinder were measured or predicted. It was done for varying temperatures, air velocities and humidity.

Many models have been published to predict frost growth. These may be divided into empirical and theoretical models. All prior theoretical models assume the water vapor is saturated at the interface between the air stream and frost layer, (Na and Webb (2004))

Schneider (1978) proposed the correlation of frost thickness on the circular tube, but the effect of air velocity is neglected in the correlation. Ostin and Andersson (1990) suggested the correlations for the thermal conductivity of the frost layer as a single function of frost density on a cold plate. Yang and Lee (2004) proposed the correlation for the frost layer thickness and the dimensionless frost density on a cold plate. Recently Barzanoni et al (2012) proposed in study, dimensionless correlations based on previous experimental data and reported empirical correlations. The frost conduction coefficient is determined by using an analytical equation.

Sengupta et al. (1998) proposed empirical heat transfer and frost thickness correlations during frost deposition on a cylinder in humid air cross-flow. Mago and Sherif (2003) presented a semi-empirical model describing heat and mass transfer on a cylinder surface in humid air cross flow under a supersaturated frosting condition. Kim et al. (2008) proposed dimensionless correlations of frost properties on a cold cylinder surface. Frosting experiments were performed while changing various frosting parameters such as the air temperature, cold cylinder surface, air velocity, and absolute humidity. Dimensionless correlations for the thickness, density and surface temperature of the frost layer, and the heat transfer coefficient were obtained as function of the Reynolds number, Fourier number, absolute humidity, and dimensionless temperature.

Some of the empirical correlations for the prediction of frost thickness ($\delta_f$) and Nusselt number (Nu) that has been presented by previous researches are summarized in Tab.1.
Therefore, a numerical study was realized and experimental comparisons for the validation of the model.

2 FORMULATION AND THE SOLUTION OF THE FROST FORMATION PROCESS

The model the first phase in the frost formation process is to consider the one dimensional crystal growth as in Fig. 1 and write the energy and mass balance for the element of the form:

\[ \rho c_p s \frac{dT_f}{dt} = 4h(T_f - T_p) + 2k \frac{\delta d T_f}{\delta r} + k \frac{\delta^2 T_f}{\delta r^2} + 2 \rho \frac{\delta d}{\delta r} \]
When the first terms in right-hand side denoted convection heat transfer with ambient, second and third terms denotes conduction heat transfer in the element across r direction, and the last term in right-hand denotes the heat transfer due to phase change from vapor to ice.

\[
p\frac{\partial d}{\partial t} = 2h_m(W_Y - W_\beta)
\]  

\[ (2) \]

![Figure 1. General form of the ice crystal](image1)

![Figure 2. Elementary volume \(\Delta V\) in the porous frost layer](image2)

The boundary and initial conditions for this stage are:

\( r = \delta \): where the process of mass transfer is taking place and initial crystal diameter is \( d_0 \):

\[
d = d_0.
\]

\[ (3) \]

\[
d = d_0;
\]

\[ (4) \]

\[
pd \frac{\partial \delta}{\partial t} = h_m d_0 (W_\infty - W_s) \Rightarrow \frac{\partial \delta}{\partial t} = \frac{h_m}{\rho} (W_\infty - W_s) - \rho \lambda d_0 \frac{\partial \delta}{\partial t} = k d_0 \frac{\partial \delta}{\partial t}.
\]

\[
h d_0 (T_\infty - T_s) \Rightarrow \frac{\partial T}{\partial r} = \frac{h}{k} (T_\infty - T_s) + \frac{\rho \lambda}{k} \frac{\partial \delta}{\partial t}.
\]

\[ (5) \]

\[
r = 0; T = T_s; \frac{\partial d}{\partial r} = 0.
\]

\[ (6) \]

The initial conditions

\[
\delta_f(t = t_{tr1}) = \delta_f(0)
\]
\[ d(r, t = t_{n+1}) = d_0 \]
\[ T(r, t = t_{n+1}) = T_c \]

During the solution of the first stage many numerical tests were realized to optimize the computational grid and also to validate the model comparing the predictions with available experimental and numerical results. These comparisons will be presented later during the discussion of the results.

It is important to notice that \( T_\gamma \) is a function depending on \( \alpha \) which varies between zero when the Reynolds number (Re) is very small and 1.0 when the Reynolds number are very high and can be written as:

\[ T_\gamma = \alpha T_\infty + (1 - \alpha) T_\beta \quad (7) \]

In order to continue the solution towards the second stage is necessary to use the results from the first stage as coupling conditions for the second stage.

Considering the subdivision \( i \), we can write:

\[ \epsilon_\beta i = \frac{\pi d^2}{4 i^2} \quad (8) \]
\[ T_i = \epsilon_\beta i \rho_\beta + (1 - \epsilon_\beta i) T_\gamma \quad (9) \]

This procedure is recommended by Tao et al.(1992) and Ismail et al.(1997). The values of \( \rho_\beta i \) and \( c_p \) are determined in terms of \( \epsilon_\beta i \) and \( T_i \) in the second stage.

In the second stage the frost is fully developed and consequently can be treated as a porous layer. In the treatment of second stage we can assume that the transfer of mass and energy is one dimensional, the pressure in the porous medium is uniform, thermodynamic equilibrium exists, that is, the temperature of the liquid and solid phases are equal, diffusion process in the medium is dominant and finally the air within the porous medium is saturated.

The local volume averaging technique described by Tao et al.(1992), Gall and Grillot (1997) and Ismail et al.[5] will be used. Here, it consists in considering an elementary volume \( \Delta V \) in the porous frost layer as shown in Fig.2, where both \( \beta \) (ice) and \( \gamma \) (moist air) phases coexist. Therefore we can write the energy and mass equation for the \( \gamma \) and \( \beta \) phases as:

\[ \rho \cdot c_p \frac{\partial T}{\partial t} + \lambda \cdot \nabla \cdot \left( k_{eff} \nabla T \right) = \frac{\partial}{\partial r} \left( k_{eff} \frac{\partial T}{\partial r} \right) \quad (10) \]
\[ \frac{\partial \epsilon_\beta}{\partial t} + \frac{\nabla}{\rho_\beta} = 0 \quad (11) \]
\[ \frac{\partial}{\partial t} \left( \epsilon_\gamma \rho_\gamma \right) - \nabla \left[ D_{eff} \nabla \rho_\gamma \right] = \frac{\partial}{\partial r} \left( D_{eff} \frac{\partial \rho_\gamma}{\partial r} \right) \quad (12) \]

The initial conditions are obtained from the coupling conditions, while the moving interface conditions are:
Before we can initiate the simulation of frost formation, it is necessary to determine the general thermo-physical properties of the different phases and the transfer coefficients using experimental and empirical correlations available in the literature.

The thermal conductivity (Dietenberger [1983]):

$$k_B = \frac{630}{T} \quad (W/m.K)$$

The specific heat, density and thermal properties of the dry

$$c_{pB} = 2116.56 + 7.2845T \quad (J/kg°C)$$

$$\frac{1}{\rho_B} = 0.0010907 + 1.4635 \times 10^{-7}T \quad (kg/m³)$$

The thermal properties of the dry are determined from:

$$k_{ar} = 0.001968 + 8.15 \times 10^{-5}T \quad (W/m.K)$$

$$c_{par} = 1004 \quad (J/kg°C)$$

$$\rho_{ar} = \frac{344.9}{T} \quad (kg/m³)$$

The effective thermal conductivity of the porous frost layer is evaluated as

$$k_{eff} = 0.02422 + 7.214 \times 10^{-4} \rho_f + 1.1797 \times 10^{-6} \rho_f^2 \quad (W/m.K)$$

The effective diffusion coefficient is estimated:
\[ D_{eff} = \varepsilon_y D (1 + F) \] (m²/s) \hspace{1cm} (24)

where:

\[ D = 2.19 \left( \frac{T}{T_0} \right)^{1.81} \times 10^{-5} \] (m²/s) \hspace{1cm} (25)

The effective diffusion coefficient at the surface is:

\[ D_{eff} = \varepsilon_y D (1 + F_s) \] (m²/s) \hspace{1cm} (26)

\( F \) and \( F_s \) are as given by Tao et al. [1992].

The relative humidity is estimated from:

\[ w = 0.6218 \frac{P_{v,sat}}{P_{total} - P_{v,sat}} \] \hspace{1cm} (27)

Where the saturation pressure over ice at any instant and any point is estimated from the empirical equation of ASHRAE [2009] valid for the range 123.15K<T<273.15K

\[ \ln (P_{v,sat}) = \frac{C_1}{T} + C_2 + C_3 + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T \] \hspace{1cm} (28)

where:

\[ C_1 = -5.674 535 9 E+03 \quad C_2 = 6.392 524 7 E+00 \]
\[ C_3 = -9.677 843 0 E-03 \quad C_4 = 6.221 570 1 E-07 \]
\[ C_5 = 2.074 782 5 E-09 \quad C_6 = -9.484 024 0 E-13 \]
\[ C_7 = 4.163 501 9 E+00 \]

The sublimation latent heat is estimated from the equation due to Parish [1970]:

\[ h_{lg} = (-0.04667 \times (1.8(T - 273) + 32) + 1220.1) \times 2322 \] (J/kg) \hspace{1cm} (29)

Values of the properties used in the non-dimensional equations are given by Tao et al.[4]:

\[ \alpha_{eff} = 1.38 \times 10^{-6} \] (m²/s) \hspace{1cm} (30)
\[ \rho_{fo} = 92.84 \] (kg/m³) \hspace{1cm} (31)
\[ k_{eff} = 0.245 \] (W/m.K) \hspace{1cm} (32)
\[ c_{p,fo} = \frac{k_{eff}}{\alpha_{eff} \rho_{fo}} = 912.28 \] (J/kg.K) \hspace{1cm} (33)
\[ \delta_p = 0.029 \text{ (m)} \] (34)

### 2.1 Empirical Nusselt correlation estimation

To predict frosting process formation in a cylindrical surface a numerical solution based on the formulation of the local volume averaging technique it was utilized. Numerical simulations for frost grows using the correlation of experimental due to Kim [2008], Eq. (35) were realized, but the comparison with this empirical correlations of frost grow is bad.

\[ \text{Nu} = 0.437 \cdot \text{Re}^{0.431} \cdot \text{Fo}^{0.275} \cdot \text{wo}^{-0.173} \cdot \left( \frac{T_{oo}D}{T_{tp}D} \right)^{-9.61} \cdot \left( \frac{T_{c}D}{T_{tp}D} \right)^{7.221} \] (35)

Based on the correlation experimental Eq. (35), a new correlation for Nusselt number was estimated.

\[ \text{Nu} = k \cdot 0.437 \cdot \text{Re}^{0.435} \cdot \text{Fo}^{-0.275} \cdot \text{wo}^{-0.173} \cdot \left( \frac{T_{oo}D}{T_{tp}D} \right)^{-9.661} \cdot \left( \frac{T_{c}}{T_{tp}D} \right)^{7.221} \] (36)

For this new correlation an optimization method that adjusts the numerical solution of modeling the frost formation process with experimental results of the frost layer thickness was used. The calculation procedure allows the estimation of the parameter K of equation. The eq. (36) was used to find the value of K that approximates the numerical solution to the empirical correlation for frost grow due to Kim [35]. The K value is approximate using the comparison of k values that super estimate and sub estimate the frost grow using the following relation refer to fig 3:

![Figure 3. The calculation procedure of K](image)

\[ k = \frac{k^+ + k^-}{2} \] (37)
3 RESULTS AND DISCUSSION

The finite difference approximations of Eqs. 1, 2 and 10–12 are obtained using the implicit approach with the upwind difference scheme for the time derivative, the central difference for the internal nodes, and the backward or forward difference for the boundary nodes. The numerical study is initiated by optimization of grid size and time increment used in the numerical simulations. The numerical results of the first stage are presented in terms of the dimensionless ice diameter for different properties as shown in Fig. 4. Various numerical tests were realized to optimize the numerical parameters such as the time and the space tests. Fig.5 shows the effect of varying the time interval for the cases 1, 2 and 4s. As can be see the results seem to be coincident. The spatial grid in a radial direction uses 20 intervals on a frost thickness.

Numerical simulations were realized with the empirical correlations for the Nusselt number, using eq. (36) and (37) and it is calculate the K number as equal to 1.6. Figures 6 to 11 show results of simulations of frost grow using eq. (36) for different values of air velocity, air humidity, air and surface temperature and it is compared with the empirical correlation due to Kim [35]. Figure 12 shows comparison with simulation using a Nu correlation due to Sengupta [8]. The ranges of applicability of the equation (36) are: 0, 5 m/s ≤ air velocity ≤2 m/s and 253K ≤ Tc ≤268 K. Figures show good agreement with new correlation of experimental and numerical predictions. Realizes that the agreement is best for temperatures of 260 K and velocity 0.5 m/s. Figure [13] show the density of the frost layer.
Figure 6. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=0.5 \, \text{m/s}, \, T_c=260\,\text{K} \)

Figure 7. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=0.5 \, \text{m/s}, \, T_c=268\,\text{K} \)

Figure 8. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=0.5 \, \text{m/s}, \, T_c=258\,\text{K} \)

Figure 9. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=0.5 \, \text{m/s}, \, T_c=253\,\text{K} \)

Figure 10. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=1.0 \, \text{m/s}, \, T_c=260\,\text{K} \)

Figure 11. \( w_0 \cdot 0.01 \, \text{kgv/kgar}, \, \text{vel}=2.0 \, \text{m/s}, \, T_c=260\,\text{K} \)
CONCLUSION

This study uses a numerical model based in a local volume averaging technique for governing equations formulation and empirical correlations for the convective heat and mass transfer on frost formation phenomena. The effect on frost thickness grows prediction of the utilization of different empirical correlations of the Nusselt number taken from the bibliography was investigated. Numerical simulations and comparisons with available experimental data were made. Also, the effect of the other parameter as humidity, surface temperature and velocity were numerical predicted and compared with experimental data. The correlations used in the study are expressed as functions of the Reynold number, dimensionless time, humidity and dimensionless temperature. The numerical results obtained seem to be coherent and presents a good agreement with experimental data.

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